Alligation Rule and Mixtures and Replacements

Alligation rule helps us to find, in what ratio two mixtures with different concentrations are to be mixed to get a target concentration.

Assume that in an engineering college ECE branch average aggregate is 80% and that of CSE is 68%. In what ratio the students in these classes are to be mixed to get an aggregate of 76%?

Take ECE class average as A_1, CSE class average as A_2. If there are n_1 students in ECE and n_2 students in CSE then the overall average A_{avg} is calculated as follows

$$\frac{n_1 \times A_1 + n_2 \times A_2}{n_1 + n_2} = A_{avg}$$

If we re arrange this equation

$$n_1 \times A_1 + n_2 \times A_2 = A_{avg} \times (n_1 + n_2)$$

$$n_2 \times (A_2 - A_{avg}) = n_1 \times (A_{avg} - A_1)$$

$$\frac{n_1}{n_2} = \frac{(A_2 - A_{avg})}{(A_{avg} - A_1)}$$

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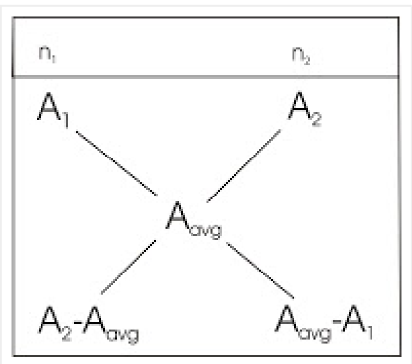
To apply this rule easily we follow a small diagram

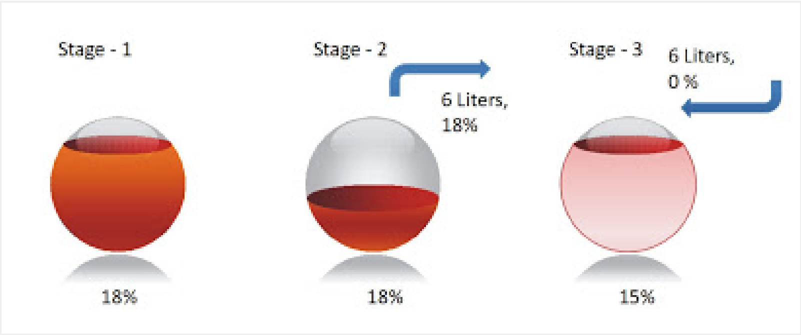


Even though alligation rule is widely applied in mixtures, we can see this rule in many areas across arithmetic.

Competitive exam is where you need to solve problems in extreme time constraints. So you need to develop clear understanding of concepts to solve problems quickly. Let us discuss a classic problem to understand how solid concepts enable us to clear the tests easily.

A cask contains alcohol solution with concentration 18%. If 6 Liters of this mixture is taken out and replaced with water, then concentration drops to 15%. What is the original volume in the cask.

Explanation:



You can observe from the diagram, concentration remains the same after 6 liters were removed from the cask. Concentration changes inversely proportional to the volume of water added to the mixture. So change in the concentration happened only from second stage to third change.

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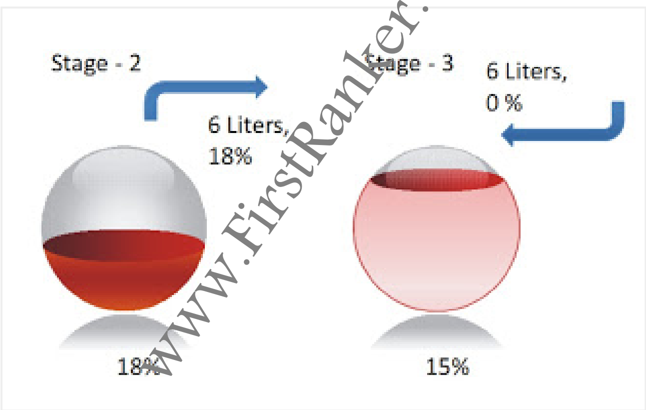


Method 1:

We know that total alcohol component in the mixture is equal to alcohol component in the mixture that has been taken out plus remaining alcohol in the cask. so assume initial or final volume is V liters.

18% (V) = 18% (6) + 15% (V)

3% (V) = 18% (6)

V = 36

Method 2:

Volume and concentration are inversely proportional to each other.

IV = initial volume; IC = initial concentration; FV = Final volume; FC = Final concentration

IV x IC = FV x FC

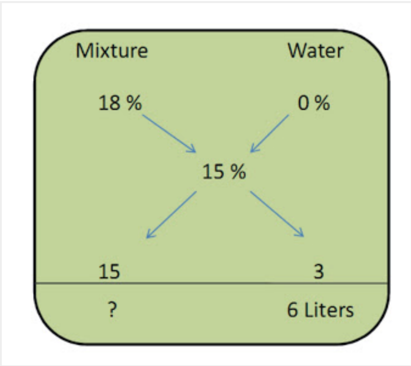
(V-6) x 18% = V x 15%

(V-6) / V = 6 / 5

So V = 36

Method 3:

We can also solve this problem by using alligation rule. It states that in what ration two components are mixed to get a targeted concentration. We can apply this rule to this problem for the second stage. We added water which is at 0% concentration to a mixture of 18% concentration to get a solution with 15% concentration.



So we understand from the above diagram Mixture and water should be mixed in the ration 15:3 to get desired concentration 15%. But we know that 3 units of water is equal to 6 liters so 15 units of mixture is equal to 30 liters. Total volume is equal to 30 +6 = 36. Please note that in the second stage the volume is equal to (V - 6)

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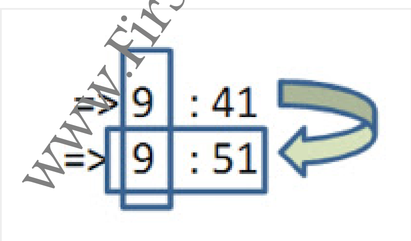
Method 4:

Initial Condition 18 % = (A : W) = 18 : 82

Final Condition 15 % = (A : W) = 15 : 85



We know that there is no change in the Pure alcohol component from second stage to third stage. so we can equate alcohol component in the above two equation by multiplying with appropriate numbers. Now we observe a change in the water components from 41 to 51. This is due to the water we added to the mixture. We added 6 liters of water which is equal to 10 units change in the mixture. so

10 Units = 6 Liters

60 Units = 36 Liters

Method 5:

We can use this formula

$$\Rightarrow FC = IC\left(1 - \frac{x}{v}\right)^{n}$$

$$\Rightarrow 15\% = 18\%\left(1 - \frac{x}{v}\right)^{n}$$

Here n = 1 because we made this substitution only once.

$$\Rightarrow \frac{15}{18} = \left(1 - \frac{6}{v}\right)$$

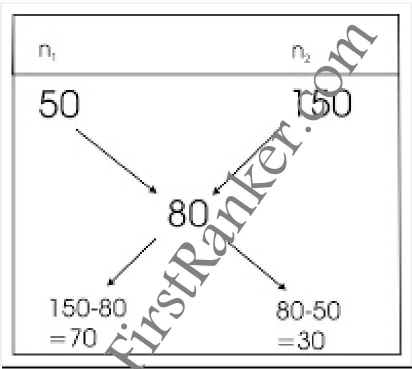
$\Rightarrow \frac{5}{6} = 1 - \frac{6}{v}$
$\Rightarrow \frac{6}{v} = 1 - \frac{5}{6}$
$\Rightarrow v = 36$

Practice Problems

1. A person covers 800km partly at a speed of 50kmph and partly at a speed of 150kmph, in 10 hours over all. What is the distance covered at the speeds of 150 kmph?

His average speed for the entire journey is $\frac{800}{10} = 80$

Now we need to find in what ratio he needs to travel 800km, partly at 50kmph, and 150kmph to get average speed of 80kmph



So for every 70 parts of the time travelled at 50kmph, he has to travel 30 parts are 150kmph.

As the total time is 10 hours he must have travelled $\frac{30}{70+30} \times 10$ = 3 hours at 150 kmph.

So distance travelled = 150 × 3 = 150

2. The cost of oil is Rs.100 per kilogram. After adulteration with another oil, which costs Rs.50 per kilogram, Ram sells the mixture at Rs.96 per kilogram making a profit of 20%. In what rate does he mix the two kinds of oil?

Before applying the alligation rule we need to make sure that all the parameters are in the same units. Here 2 costs prices and 1 selling price was given. So we should convert the selling price into cost price.

Given $C.P \times (100+20)\% = 96$

$$\Rightarrow C.P = \frac{96}{120\%} \quad \Rightarrow C.P = \frac{96}{\frac{120}{100}} \Rightarrow 96 \times \frac{100}{120} = 80$$

Now we apply the alligation rule.

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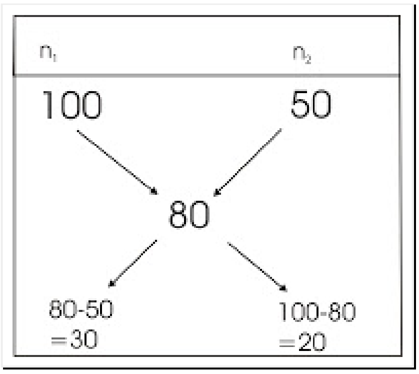

So These two must be mixed in the ratio 3:2.

3. Two solutions of milk and water are combined in the ratio 2:3 by volume. The resultant solution is a 40% milk solution. Find the milk concentration in the first solution if the concentration of milk in the second is 60%?

It was given that $n_1 : n_2$ are in the ratio 2:3 and second solution concentration is 60 and resultant solution concentration is 40.

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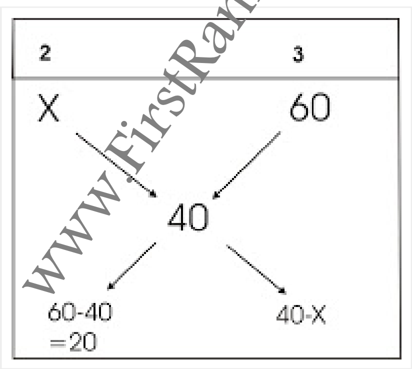
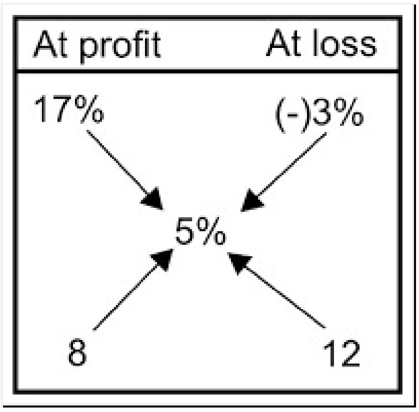


From the above we know that 20, 40 - x must be in the ratio 2: 3 $\Rightarrow \frac{20}{40-x} = \frac{2}{3} \Rightarrow x = 10$

4. A shopkeeper sold 45 kg.of goods. If he sells some quantity at a loss of 3% and rest at 17% profit, making 5% profit on the whole, find the quantity sold at profit.

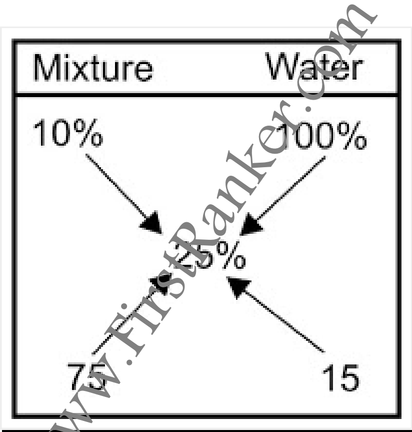


Note: 17 - 5 = 12, and 5 - (-3) = 5 + 3 = 8

Therefore, Ratio of quantities sold at profit and at loss = 8 : 12 = 2 : 3

Therefore, Quantity sold at profit = $\frac{2}{5}$ x 45 = 18 kg.

5. A mixture of 70 litres of wine and water contains 10% water. How much water should be added to make 25% water in the resulting mixture?



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Therefore, The ratio is 75 : 15 = 5 : 1.

Therefore, For every 5 litres of mixture, 1 litre of water is added.

Therefore, For 70 litres, water to be added = $\frac{1}{5}$ x 70 = 14 litres.

Alternative Method:

Water = 10% of 70 litres = 7 litres

Wine = 70 - 7 = 63 litres

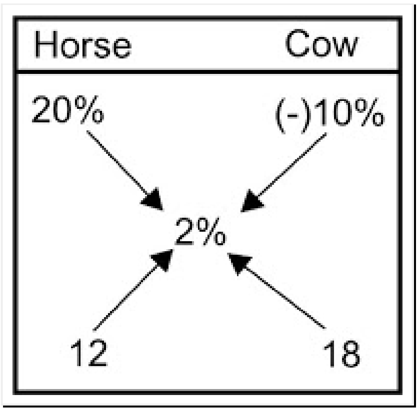
Now, in the new mixture, water is 25% and wine is 75%.

Hence, in new mixture wine is 3 times of water.

Therefore, Water in new mixture = $\frac{1}{3}$ x 63 = 21 litres

Therefore, Water to be added = 21 - 7 = 14 litres

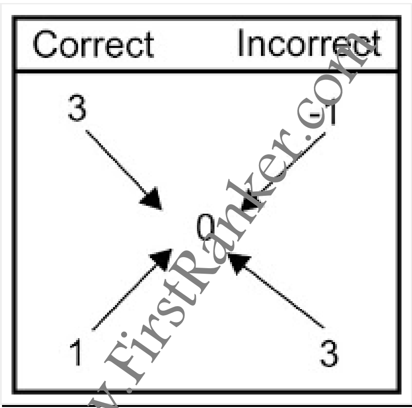
6. A man purchased a horse and a cow for Rs. 5000. He sells the horse at 20% profit and the cow at 10% loss. If he gains 2% on the whole transaction, the cost of the horse is:



Therefore, Ratio between cost of a horse and that of a cow = 12 : 18 = 2 : 3.

Therefore, Cost of the horse = $\frac{2}{5}$ x 5000 = Rs. 2000

7. In an examination, a student gets 3 marks for every correct answer and loses 1 mark for every wrong answer. If he scores '0' marks in a paper of 100 questions, how many of his answers were correct?



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Therefore, Ratio of correct and wrong answers = 1 : 3

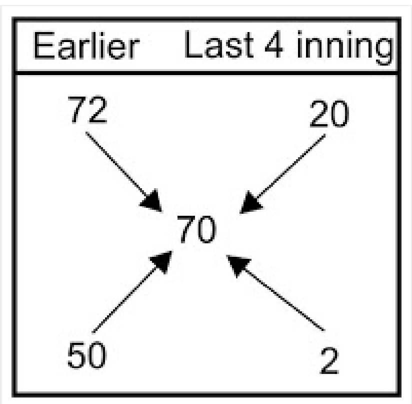
Therefore, Correct answers = $\frac{1}{4}$ x 100 = 25

8. The batting average of a cricket player is 72 runs per inning. In the next 4 inning, he could score only 80 runs and thereby decreases his batting average by 2 runs. What is total number of inning played by him till last match?

Average of last 4 inning = $\frac{80}{4}$ = 20 runs

Average of inning played earlier = 72 runs

New average = 72 - 2 = 70 runs



Ratio of inning played (before and after) = 50 : 2 = 25 : 1

Given, innings played (after) = 4

Therefore, Inning played (before) = 25 x 4 = 100

Therefore, Total inning played = 100 + 4 = 104

Alternative Method:

Average runs in the last 4 inning = $\frac{80}{4}$ = 20 runs

Short from previous average = 72 - 20 = 52

Short (total) = 52 x 4

But, short runs (per inning) = 2 runs

Therefore, Total inning played = $\frac{52 \times 4}{2}$ = 104 inning

Mixtures and Replacements

The problems related to mixtures based on two important concepts. Alligation rule and Inverse proportionality rule are the two.

In these problems we are asked to find the resultant concentration after mixing two or three components or the final concentration when one component of the mixture is being replaced by another component which is mostly one the components of the mixture.

The general formula for replacements is as follows: $FC = IC \times (1 - \frac{x}{V})^n$

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Here

FC = Final concentration

IC = Initital concentration

x = replacement quantity

V = Final volume after replacement

n = number of replacements

Note: Always remember FC and IC are the concentrations of the second component in the mixture. "x" is the concentration of the first component.

9. From a solution containing milk and water in the ratio 3 : 4, 10 L is removed and replaced by water. If the resultant solution contains milk and water in the ratio 1 : 2 then what was the amount of the original solution ?

Here also we are replacing with water. So FC and IC must be milk concentrations.

Initial concentration of the milk = 3/7

Final concentration of the milk = 1/3

Applying formula

$\frac{1}{3} = \frac{3}{7} \times (1 - \frac{10}{V})$ 1

$\Rightarrow \frac{7}{9} = 1 - \frac{10}{V}$

$\Rightarrow \frac{2}{9} = \frac{10}{V}$

$\Rightarrow V = 45$

10. 10% of a solution of milk and water is removed and then replaced with the same amount of water. If the resulting ratio of milk and water is 2 : 3, find the ratio of milk and water in the original solution.

Applying formula:

$\Rightarrow \frac{2}{5} = K \times (1 - \frac{10}{100})$

Here 2/5 is the milk concentration.

$\Rightarrow \frac{2}{5} = K \times \frac{9}{10} \Rightarrow K = \frac{4}{9}$

11. A beaker had 20 L of alcohol-glycerol mixture in the ratio 4 : 1 by volume. In the first round, 4 L of the mixture is removed and replaced with glycerol. In the second round, 5 L of the resultant solution is removed and replaced with glycerol. Finally, 10 L of the resultant mixture is removed and replaced with glycerol. What is the final quantity of glycerol in the mixture

Here we are replacing the mixture with glycerol. So we have to take Alcohol concentrations for IC and FC.

Initial concentration of alcohol is 4/5 = 80%

Applying the formula for the first replacement:

$\Rightarrow FC^1 = 80\% \times (1 - \frac{4}{20})$

Here FC^1 is the concentration after first replacement.

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Second replacement:

$\Rightarrow FC^2 = FC^1 \times (1 - \frac{5}{20})$

Third Replacement:

$\Rightarrow FC^3 = FC^2 \times (1 - \frac{10}{20})$

Now substituting the FC^1 and FC^2 in FC^3 we get

$\Rightarrow FC^3 = 80\% \times (1 - \frac{4}{20}) \times (1 - \frac{5}{20}) \times (1 - \frac{10}{20})$

$\Rightarrow FC^3 = 80\% \times \frac{4}{5} \times \frac{3}{4} \times \frac{1}{2}$

$\Rightarrow FC^3 = 24\%$

12. In a mixture of 80 L, milk and water are in the ratio 7:3. If 24 L of this mixture is replaced by 16 L if milk, find the final ratio of milk and water.

Final volume of the mixture = 80 - 24 + 16 = 72

Replacement quantity = 16

Applying formula,

$$\Rightarrow \text{FC} = \frac{3}{10} \times (1 - \frac{16}{72})$$

$$\Rightarrow \text{FC} = \frac{3}{10} \times \frac{7}{9} \Rightarrow \frac{7}{30}$$

So Milk and Water after replacement = 23 : 7

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